Public Choice **97:** 257–293, 1998.

Multiparty electoral competition in the Netherlands and Germany: A model based on multinomial probit *

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Abstract. A typical assumption of electoral models of party competition is that parties adopt policy positions so as to maximize expected vote share. Here we use Euro-barometer survey data and European elite-study data from 1979 for the Netherlands and Germany to construct a stochastic model of voter response, based on multinomial probit estimation. For each of these countries, we estimate a pure spatial electoral voting model and a joint spatial model. The latter model also includes individual voter and demographic characteristics. The pure spatial models for the two countries quite accurately described the electoral response as a stochastic function of party positions. We use these models to perform a thought experiment so as to estimate the expected vote maximizing party positions. We go on to propose a model of internal party decision-making based both on pre-election electoral estimation and post-election coalition bargaining. This model suggests why the various parties in the period in question did not adopt vote maximizing positions. We argue that maximizing expected vote will not, in general, be a rational party strategy in multiparty political systems which are based on proportional representation.

1. Introduction

Democratic political systems can be distinguished by whether they are based essentially on proportional or plurality electoral rules, and whether political parties are strongly disciplined or not (see Table 1).

Most of the polities of Western Europe have electoral systems based on proportional representation, with relatively disciplined parties. Some of these polities (such as Austria) only have two parties, but others (such as Finland) may have five or six or more. In contrast, Britain has a plurality (or first-post-the-post) electoral system, based on over 600 constituencies, each of which returns one member of Parliament. Although the House includes at

* This paper is based on research supported by NSF Grants SBR 94-22548 and 96-17708. Versions of this research have been presented at the Public Choice Meeting, San Francisco, March 1997, at the European Public Choice Meeting, Prague, April 1997, and at the Political Science Seminar, New York University, May 1997.

least eight parties, the effective number[1] is just above two (Schofield, 1997a) as would be expected from Duverger's Law (Duverger, 1954, 1984; Riker, 1982; Fedderson, 1992).

The British system is often called a "Westminster style" polity. In contrast, the U.S. Congress is based on a plurality electoral system, but (to judge from the heterogeneity of the voting by its members) the two parties are not disciplined in the sense that British parties are (Poole and Rosenthal, 1991).

In some of the new democracies (such as Russia) mixed PR and plurality electoral methods have been adopted, and the polity seems to be highly factionalized and the parties very weak. In Japan, which until recently had a complex electoral system based on multimember constituencies, the dominant liberal Democratic Party was comprised of at least six factions all of whose leaders were contenders for the prime ministerial position (Wada and Schofield, 1996).

To construct a formal comparative politics, it would be desirable to build a rational choice or game theoretic model that includes (a) a reasonable description of voters' choices, (b) a coherent description of the behavior by political actors in the pre-election context, and (c) a model of the negotiation or bargaining by political actors, in the post-election phase, over the formation of government. Presumably the type of electoral system would enter into the calculation of political actors under (b) so that it would be possible to see clearly the effect of different motivations. Similarly, the effect of political institutions (such as Congressional committees or cabinet rule) could be understood more readily, were a general model of (c) available.

Almost all the theoretical work of which we are aware has concentrated either on pre-election behavior by two candidates under plurality rule, or on post-election coalition negotiation in multiparty situations (where "multiparty" means at least three parties). It is clear enough that understanding "multiparty" competition should also involve calculations made by candidates or parties before the election. This paper will use an empirical analysis of electoral data from the Netherlands and Germany to argue that the usual two-party electoral models do not generalize well to multiparty situations. Instead we argue that a coherent model of pre-election party strategy has to incorporate both an electoral component *and* prediction over post-election coalition possibilities.

We use our model of electoral behavior, together with our account of coalition behavior, to suggest why political competition in multiparty situations does not lead to convergence of party positions. In particular, our model of party policy choice emphasizes the heterogeneity of preferred policies within each party. We argue that the proposed model can in principle be used to understand the different motivations determining political choice in electoral

Table 1. Types of politics

Discipline	Electoral rule	
	Proportional representation	Plurality
Strong	West European model	Westminster
Weak	Factional	U.S. House

systems based on proportional representation or plurality rule, with or without disciplined parties.

2. Elections: Models of pre-election choice by parties

The literature on formal models of elections is vast, but we can briefly review the main results of the spatial model using Table 2 as a guide. The spatial model assumes that political choices are points in a policy space, W, of dimension w. Each voter $i \in N$ has an ideal point, $x_i \in W$, and each candidate, $j \in K$ makes a "declaration" $\phi_j \in W$. The "electoral data" is then an $n \times k$ matrix $\delta_{ij} = || x_i - \phi_j ||$ where $n = |N|$ and $k = |K|$. In the pure spatial *deterministic* model, voter i's "utility" $\overline{u}_{ij}$ for the choice ϕ_j is a monotonically decreasing function of δ_{ij}. A typical assumption is that $\overline{u}_{ij} = -\beta\delta^2_{ij}$. Voter choice is given by an $n \times k$ matrix $X = (X_{ij})$ where $X_{ij} = 1$ (i chooses j) if $|| x_i - \phi_l || > || x_i - \phi_j ||$ for $l \neq j$, and $X_{ij} = 0$ otherwise. The share of the vote of party j is then $\pi_j(X)(\phi) = \frac{1}{n}\Sigma_{i=1}^{n} X_{ij}(\phi)$, (where $\phi = (\phi_1 ..., \phi_k)$ is the vector of declarations). In two-party deterministic models it is usually assumed that each candidate desires to win. Thus for example we could assume that the utility for candidate 1, given X and ϕ, is

$$\begin{aligned} U_1(x)(\phi) &= 1 \text{ if } \pi_1(X)(\phi) > \pi_2(X)(\phi) \\ &= 0 \text{ if } \pi_1 = \pi_2 \\ &= -1 \text{ if } \pi_1 < \pi_2. \end{aligned}$$

In the Hotelling (1929) and Downs (1957) model, vote maximizing by the candidates (in the case $w = 1$ and $k = 2$) forces each candidate (j) to adopt the electoral median (or Nash equilibrium) position, $\phi^*_j = x_{\frac{1}{2}(n+1)}$, where the voter ideal points are ordered $x_1 \leq x_2 \leq ... x_n$, etc. In the one-dimensional case this result is robust. Thus, if each candidate has policy preferences (so that 1, for example, prefers to win with a policy near some bliss point z_1, say), then

Table 2. Models of election

Candidate model	Voter model	
	Deterministic	Probabilistic
Office seeking	Low dimension Electoral median High dimension Electoral heart	Strong convergence Nash equilibrium at electoral mean, when variance is high.
Policy-seeking	Weak convergence but possibly to Nash equilibrium	Weak convergence for some parties; possible divergence by small parties

the Nash equilibrium typically satisfies $\phi^*_1 = \phi^*_2$ (Osborne, 1995). The logic is clear: to implement a desired policy, a candidate still must win. Attempts to extend the Hotelling-Downs model to the case with $k \geq 3$, generally find no pure Nash equilibrium (satisfying $\phi^*_j \in W$ for all $j \in K$), even in one dimension (Osborne, 1993).

The motivation of the candidates in such a multiparty model is usually taken to be plurality maximization, so candidate j prefers that declaration, ϕ_j, which maximizes the differences between π_j and π_l, for all $l \neq j$. The reasoning underlying this assumption is presumably that π_j is a proxy for the power of the political agent, j, and that such an agent wishes to maximize its power. We shall say that an agent, j, who acts to maximize π_j (subject to some model X of voter behavior) is *Downsian*. A theme of this paper is that Downsian behavior need not be "rational" in a more general context.

Attempts to extend the Hotelling-Downs model, with $k = 2$ but $w \geq 2$ ran into the well-known difficulty of "generic" non-existence of a pure strategy Nash equilibrium. (In fact, the formal proof of this comes from results by McKelvey and Schofield, 1986; and Saari, 1997, on non-existence of a core in a spatial committee game. We mention these results in the next section.) This "fact" can be side-stepped either by focusing on mixed strategy Nash equilibria (Kramer, 1978), or by introducing a more general notion such as the "uncovered set" (McKelvey, 1986; Cox, 1987). However, the uncovered set is a concept based on spatial committee voting theory, and it is not entirely obvious that it is appropriate for modeling elections, even when political agents are assumed to be Downsian. Below we shall introduce the notion

of the electoral and political "heart" (Schofield, 1995a), and argue that it can be used to understand multiparty competition.

Existence of pure strategy Nash equilibria with Downsian candidates is much easier to demonstrate when the electoral model (X) is probabilistic or "stochastic" (see, e.g., Hinich, 1977; Enelow and Hinich, 1984; Coughlin, 1992). There are a number of varieties of such a model, but they all suppose voter behavior is described by the probability χ_{ij} that voter i chooses j. Thus $\chi_{ij} = \text{Prob}(X_{ij} = 1)$, so $\Sigma_{j\epsilon K}\ \chi_{ij} = 1$, for each i. A typical assumption is that $u_{ij} = \overline{u}_{ij} + \epsilon_j$ where ϵ_j is a "perceptual" error term associated with candidate j and $\overline{u}_{ij}$ is the spatial utility of i for j. Usually the entries in the error vector $\epsilon = (\epsilon_1,...,\epsilon_k)$ are assumed to be independently and identically distributed (iid). The probability χ_{ij} is then the probability that $u_{ij} > u_{il}$ for all $l \neq j$. This condition is simply that

$$\epsilon_l - \epsilon_j < \beta(\delta_{il}^2 - \delta_{ij}^2) \text{ for all } l \neq j. \tag{1}$$

If it is assumed that each ϵ_j is normally distributed, with zero expected value and variance $\sigma^2{}_j$, then it is possible to compute the distribution of the random variable $\pi_j(X)(\phi)$, given the electoral model and the vector of declarations. Since the expectation of X_{ij} is simply χ_{ij} it follows that the expectation of the vote share is simply given by $E(\pi_j)(_\phi) = \frac{1}{n}\Sigma\ \chi_{ij}(\phi)$. As in the deterministic case, we shall say that a candidate, or agent j, who chooses ϕ_j to maximize $E(\pi_j)(\phi)$, subject to $(\phi_1, ..., \phi_{j-1}, \phi_{j+1},...,\phi_k)$, is *Downsian*.

In two-party competition a more plausible assumption for party 1 say, would be to maximize the probability that $\pi_1(\phi_1,\phi_2)$ exceeds $\pi_2(\phi_1,\phi_2)$. Nonetheless, under the independence assumption, these two motivations are effectively identical (Aranson, Hinich, and Ordeshook, 1974). A standard result for k = 2 is that a pure-strategy Nash equilibrium exists and is *strongly convergent*, in the sense that $\phi_1^* = \phi_2^*$, both at the mean of the voter ideal points. As Lin, Enelow, and Dorussen (1996) have recently shown, even when $k \geq 3$ there is a pure strategy Nash equilibrium at the voter mean (at least in the case the variance terms σ_j^2, $j \epsilon K$, are sufficiently high). In fact, they show (for high variance) that each expectation $E(\pi_j)(\phi)$ is concave in ϕ_j. Thus the Nash equilibrium is unique, and is at the "welfare maximum" point, ϕ^* (so $\phi_j^* = \phi^*\ \forall\ j \epsilon K$). That is to say, $\Sigma_{i\epsilon N} || x_i - \phi^* ||$ is minimized at the convergent Nash equilibrium, ϕ^*. However, for low variance, $E(\pi_j)(\phi)$ may fail concavity or quasi-concavity. Indeed, Nash equilibria may be divergent or fail to exist.

It would appear that both deterministic and probabilistic models with Downsian agents are preoccupied with finding convergent Nash equilibria. This seems strange, since in no political system of which we are aware do candidate positions display strong convergence. It is true that one can weaken the degree of convergence in 2-candidate elections, by assuming that candi-

dates have policy preferences. This is done by assuming that the outcome of a pair of declarations, (ϕ_1,ϕ_2), is a lottery, {(Prob(π_1),ϕ_1),(Prob(π_2),ϕ_2)} say, where Prob(π_j) is the probability that candidate j wins the election. As Cox (1984) has shown, if each candidate (or party) has a Euclidean preference based on the party's preferred point, and is committed to the implementation of its declared position if it wins, then there is a (non-convergent) pure strategy Nash equilibrium.

As soon as parties have preferred positions, then the validity of the assumption of policy commitment in Downsian models becomes suspect. We shall come back to this question later.

3. Committees: Models of post-election coalition behavior

In two-candidate elections the formal analysis is over when the election is over and one party wins. If the candidates are parties and $k \geq 3$, then the process of government formation involves creating a coalition (which may or may not be winning). The early work on European coalition governments was much influenced by Riker's (1962) analysis of constant sum voting games and his notion of minimal winning coalitions. It was soon noticed (Herman and Pope, 1973) that coalition governments could be minority (lacking a majority) or surplus (with extra partners). This research program then focussed on coalition negotiation between parties with preferred policies in a one-dimensional policy space (Axelrod, 1970; de Swaan, 1973). Essentially, these models implied that the party at the median "legislature" policy would belong to the government. In fact, one could go further and argue that if parties were *only* concerned with policy, then any party at the median position in the legislature could form a minority government and implement its desired policy. However, empirical analysis (Taylor and Laver, 1973) suggested that these one-dimensional models did not provide a satisfactory explanation of government formation.

During the 1980s however, theoretical work on the spatial "committee" model of voting (Schofield, 1985) as well as empirical work on party declarations (Budge, Robertson, and Hearl, 1987) suggested that a two-dimensional analysis of coalition behavior could be fruitful.

To illustrate, Table 3 presents the election results of the 1977 and 1981 elections in the Netherlands.

A coalition of the CDA and VVD formed in December 1977, controlling 77 seats, and lasted 41 months until the 1981 election. (It should be noted that this coalition took 6 months to form after the May 1977 election.) After 1981, a brief "surplus" coalition of PvdA, D66 and CDA (with 109 seats) formed

Table 3. Elections in the Netherlands, 1977 and 1981

Party (acronym)	1977	1981
	(Seats)	
Labor (PvdA)	53	44
Democrats '66 (D66)	8	17
Liberals (VVD)	28	26
Christian Dem Appeal (CDA)	49	48
	(138)	(135)
Communists (CPN)	2	3
Dem '70 (D70)	1	0
Radicals (PPR)	3	3
Pacific Socialists (PSP)	–	3
Reform Federation (RPF)	–	2
Reform Pol Ass (GDV)	1	1
Farmers Party (BP)	1	0
State Reform Party (SGP)	3	3
	(11)	(15)
Total	149	150

and then broke down to a minority, D66, CDA coalition. A new election had to be called in September 1982.

As described in Laver and Schofield (1990), if the parties do have policy preferences, and if the space is essentially two-dimensional, then it is possible for there to be a single "dominant" party at the "core" position. A "core" is a preferred policy position, z_1, for party 1 say, such that no other position, y say, is preferred by a coalition commanding a majority of the seats. Attempts to use the spatial committee model have foundered on the difficulty of estimating party positions. The work presented in Budge, Robertson and Hearl (1987) attempted to use content analysis of party declarations (manifestos) to estimate party positions. However these estimated positions seemed excessively volatile. In this paper we have used the Euro-Barometer II data (Rabier and Inglehart, 1981) and the European Political Parties Middle Level Elites data (ISEIUM, 1983) to estimate party positions (further discussion of these data is provided in the next section, and in Appendix A. See also Quinn et al., 1996.) These data allow us to represent the positions of the four major

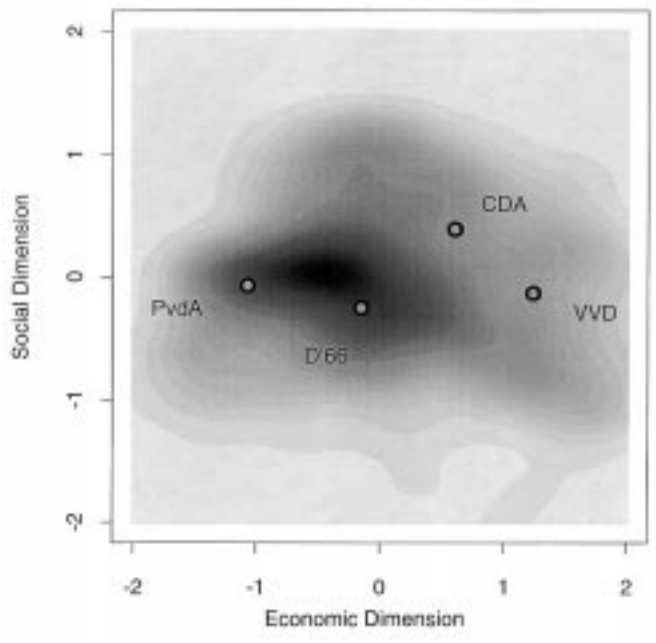

Figure 1. Dutch voter and party positions, 1979.

parties in the Netherlands in 1979 in a two-dimensional policy space, W (see Figure 1).

Assume for the moment that the four positions in this figure are preferred policy choices for the four parties and that each party, j, has a Euclidean policy preference derived from a utility function of the form $U_j(x) = -||x-z_j||^2$ (where z_j is its preferred position). Now draw the contract curves between the three pairs {CDA,VVD}, {PvdA,VVD} and {PvdA,CDA}. Using the election results of 1977 to determine seat strength, it is very readily shown that the "core" is empty. To illustrate, the CDA position in Figure 1 can be beaten by a winning coalition of {PvdA,VVD} adopting a position on their contract curve which is nearer to their preferred positions. This in turn can be beaten by a position on the {PvdA,CDA} contract curve and this again can be beaten by a position on the contract curve of {CDA,VVD}. The three contract curves bound a triangle {CDA,VVD,PvdA} which has been termed the political "heart" (Schofield, 1993).

It has recently been shown (Schofield, 1997b) that the "heart" $\mathcal{H}$ can be identified with a local version of the uncovered set. If the heart is viewed as a correspondence $\mathcal{H}$, from the space of all party declarations and voter behavior, to the policy space, W, then $\mathcal{H}$ is lower hemi-continuous. Thus $\mathcal{H}$ admits a continuous selection (Schofield, 1995a). See Appendix C for details.

To be more specific, given a vector of party positions, ϕ, and post-election weights (seats), we assume that the coalition outcome is a *lottery g*, namely a set of government coalitions and associated coalition probabilities, g(X)(ϕ), determined by ϕ and the stochastic electoral model X. Let $\mathcal{H}$(X)(ϕ) be the realization of the heart once the electorate has responded to ϕ, and let $\tilde{\mathcal{H}}$ denote all lotteries over $\mathcal{H}$. Then our formal model assumes that the political outcome, g(X)(ϕ), belongs to $\tilde{\mathcal{H}}$(X)(ϕ). As ϕ varies, then g is a continuous selection from $\tilde{\mathcal{H}}$. If we impute preferences to political agents then, knowing g, we may solve the implicit game, g, to deduce Nash equilibria, ϕ^*, to this game. This is a model of high generality, but of little applicability if we cannot model party preferences or the electoral response, X, appropriately.

To develop the Dutch example resulting from the 1977 election, we shall continue with the assumption that the parties are committed after the election to the various positions in Figure 1, and also suppose that coalitions form to propose policy points inside the realized heart. Without a core in 1977 the parties found great difficulty reaching an agreement. In particular the PvdA and CDA could not find an acceptable compromise. As we have observed, after months of negotiation the CDA and VVD eventually formed a minimal winning coalition government.

We see this as providing some degree of empirical justification for the heart. To pursue the example, we can chart the change in the heart resulting from the 1981 election. As Table 3 makes clear, the gain of 9 seats by D66 meant that the {CDA,VVD} coalition lost its majority (dropping to 74 seats). The heart now contracts to become the triangle {PvdA,CDA,D66}. It is hardly surprising that this three-party coalition first formed. Nor is it surprising that it collapsed to the minority {CDA,D66} coalition. The rivalry between the CDA and PvdA led to a political crisis, to a new election in September 1982, and to a relatively long-lived coalition of {CDA,VVD}. Observe that the very small parties in 1977 and 1981 seemed to play no significant role in coalition bargaining. For this reason we ignore their effect on elections and coalition bargaining.

Note that if only one dimension (the left-right economic axis) were relevant in the Netherlands then theory would suggest that the CDA (being typically at the one-dimensional core) could form a one-party minority government. To our knowledge, minority governments are very rare in the Netherlands and tend to be short-lived. This suggests, *contra* de Swaan (1973), that one-dimensional models of coalition behavior are inadequate, at least in understanding Dutch politics.

The concept of the "political" heart, used here to interpret these two post-election situations in the Netherlands, is clearly based on the spatial theory of committees, since the heart assumes that the party positions are preferred

policies, and uses the electoral strength of each of the parties to determine the pattern of winning coalitions. However, we can use the notion of the heart to infer something about the motivations of the parties before the elections. It is apparent that the simple maxim of increasing electoral vote is not adequate. Although the PvdA was the largest party in 1977, it could not find a coalition partner with which to form the government. It should also be observed that the principle of "maximizing electoral vote" ignores attitude to risk by the parties. In the stochastic electoral model discussed below, our estimates for the distribution of vote shares for the four parties show that there is a significant variance. For example, the 95% confidence interval for the number of seats which would have been obtained by the PvdA in an election in 1979 (as given by our estimation based on party positions) ranges from 42 to 54.

We shall argue in the next section that no party in the Netherlands could choose an acceptable policy position, in the period under discussion, so as to give it a reasonable probability of being at the core. In the absence of a post-election core, negotiations between the parties were bound to be as difficult as we have observed they were.

We can contrast this core-less example with one from Germany, also for 1979. Table 4 presents the election results between 1976 and 1982. Figure 2 gives the party positions obtained from ISEIUM (1983) data (see also Appendix A and Martin and Quinn, 1997). Clearly the three parties are almost, but not quite, colinear. If the positions were colinear, then the FDP would be at the core. However, Schofield (1986) shows that such a core is "structurally unstable", since small perturbations in party positions destroy the core property. Nonetheless, it can be inferred that the FDP is pivotal. The "grand" coalition of {CDU,SPD} was possible, but appears to have had low probability. In fact the {SDP,FDP} coalition formed in December 1976 and lasted until the October 1980 election. This was followed by a {CDU,FDP} coalition, which has persisted to the present. Although the Greens won 27 seats in 1982, it is clear that they have been effectively superfluous in coalition bargaining, at least until the present. (This may change after the forthcoming election [Summer 1998] in Germany.)

Essentially the FDP could pick and choose between the two large parties. In this simpler situation, there may be some reason for one of the two large parties to seek a core position. However to be at a "structurally stable" core in this situation means to command a majority of the seats. Our electoral model, to be introduced below, can be used to determine whether this is indeed possible.

We may also compare the two cases of an empty core (in the Netherlands) or a "structurally unstable core" (in Germany) with the possibility of a structurally stable core. As with the uncovered set, the heart and the core coincide

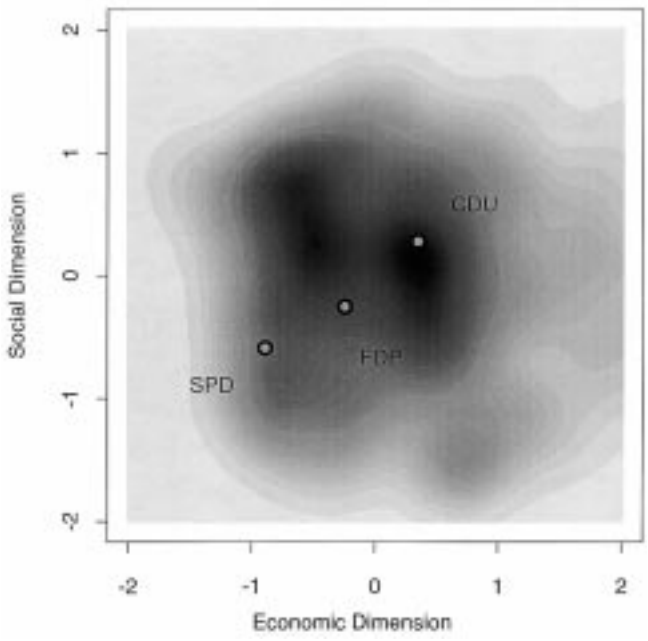

Figure 2. German voter and party positions, 1979.

Table 4. Elections in West Germany, 1976, 1980 and 1982

	1976	1980 (Seats)	1982
Christian Democrats (CDU)[a]	243	226	244
Free Democrats (FDP)	39	53	34
Social Democrats (SPD)	214	218	193
Greens (GR)	–	–	27
Total	496	497	498

[a]The Christian Social Union (CSU) seats are included with the CDU.

when the core is non-empty. The post-election bargaining theory proposed in Laver and Schofield (1990) and Schofield (1993) suggests, moreover, that in a two-dimensional policy space, if the core is non-empty and structurally stable, then it can only be occupied by the "dominant" party (see Schofield, 1995b for the proof, and a more precise definition of "dominant"). To give an indication of how a structurally stable core may arise, consider an artificial situation for the Netherlands where we suppose that the D66 is located within

the {CDA,VVD,PvdA} triangle, and controls 39 seats, with each of the other three parties controlling 37 seats. It is clear, in this case, that the D66 policy can be beaten by no other two-party coalition. The analysis provided in Laver and Schofield (1990) suggests, moreover, that such a dominant party at the core is likely to form a minority (one-party) government. In contrast, when a minority government forms that is not based on the core in this fashion, then it is nearly always short-lived. The best evidence is from Scandinavian countries: out of 50 governments in Denmark, Norway and Sweden, twenty-eight were minority, and all but three were associated with core, dominant parties. The three minority, non-core examples were all short-lived, caretaker governments.

If a party can control policy and obtain government perquisites from locating itself at a core, then the logical choice of any party would be to seek the core position. However, such a strategy is feasible only if that policy position is acceptable to the elite in the party. Note also that whether a position is core-like or not depends on the position (and strengths) of the other parties. Thus maneuvering for the core is much more difficult than simply maximizing expected vote. Indeed, there may be no core. In particular the McKelvey-Schofield-Saari results on core existence imply that the core is typically empty in weighted voting games in three dimensions.

Thus, a "counter-core" strategy by a smaller party could be to introduce policy dimensions to destroy any possibility of occurrence of a core. However, for this to be effective the new dimension must be relevant in terms of the true preferences of the parties and of the voters.

A second possibility, even in two dimensions, is that the underlying electoral response to party positions prohibits any party from advantageously locating itself at the core. To examine this possibility, we shall construct a "stochastic" model of voting for the Netherlands and Germany, based on multinomial probit (MNP).

4. An empirical model of electoral behavior based on pure spatial theory

Appendix A presents the empirical procedure we adopted to construct individual and party ideal points in the Netherlands and Germany. Because of the way the scales were constructed, a respondent agreeing strongly that "greater effort should be made to reduce inequality" would be assigned a negative value, and thus tend to be on the left on the first "economic" dimension. On the other hand, a respondent in the Netherlands agreeing strongly that "women should be free to decide for themselves in matters concerning abortion" would obtain a negative score on the second dimension. The second

dimension thus provides a technique for distinguishing Liberal voters (who might tend to vote for the VVD) and others who might tend to vote for the Christian Democratic Appeal. This second dimension in the Netherlands may be viewed as gauging beliefs on individual liberty, with a large negative value corresponding to "Libertarian" views.

The second dimension in Germany is slightly different. A respondent with a high negative value on this dimension would tend to agree strongly that "greater public control should be exercised over multinational corporations" but to disagree strongly that "nuclear energy should be developed in order to meet future energy needs." We could identify this second dimension in Germany with "corporatism." However, for convenience, we shall refer to this second dimension in both countries as a "social" dimension. The factor loadings in both countries were renormalized so that 95% of all ideal points in each dimension lay in the range [–2,+2], with the mean position at (0,0).

Note that although both "policy" dimensions in the two countries might appear to be correlated, our estimates of the underlying density functions (of voter ideal points) suggest that voter responses are not highly correlated in these dimensions. In particular, Figure 2 suggests, for example, that a German voter may simultaneously approve of egalitarian measures as well as believing that nuclear energy should be developed and multinational companies should be relatively unregulated.

By this method we obtain a "profile" $(x) = (x_i)_{i\epsilon N}$ of voter ideal positions, where each x_i represents, in some degree, the economic and social beliefs of voter i. Using almost identical questions asked of the party elites, and taking the median of these elite positions (in both dimensions) for each party, we obtain an estimate of the "profile" of party positions $\phi = (\phi_j)_{j\epsilon K}$ as given in Figures 1 and 2. Given the empirical distribution of voter ideal points, we can smooth this to approximate the underlying density function of the voters' positions. The backgrounds in Figures 1 and 2 give our estimates of these densities. The combination of (x) and (ϕ) gives a data matrix $\delta = (\delta_{ij})^{j\epsilon K}_{i\epsilon N}$. In both countries voting intentions were known, so this gives an array $y = (y_{ij})^K_N$ where $y_{ij} = 1$ iff voter i intends to vote for j, and 0 otherwise. The challenge is to estimate a probability matrix (χ_{ij}) given (δ_{ij}) and (y_{ij}), so that a realization(X_{ij}) of (χ_{ij}) is close to (y_{ij}). Following the standard assumptions of the pure spatial probabilistic model described in Section 2, we assume

$$\begin{aligned} \chi_{ij} &= \text{Prob}(u_{ij} > u_{il} \text{ for all } l \neq j) \\ &= \text{Prob}(\epsilon_l - \epsilon_j < \beta(\delta_{il}^2 - \delta_{ij}^2) : l \neq j) \end{aligned}$$

Let $e^j = (\epsilon_1 - \epsilon_j, \ldots \epsilon_{j-1} - \epsilon_j, \epsilon_{j+1} - \epsilon_j, \ldots \epsilon_k - \epsilon_j)$ be a (k–1) dimensional stochastic variable, with probability density function f and let

$$\Delta_i^j = ((\delta_{il}^2 \delta_{ij}^2), \ldots, (\delta_{ik}^2 - \delta_{ij}^2)) \epsilon \mathcal{R}^{k-1}.$$

Using Equation 1, and conditioning on our assumptions on the implicit covariance matrix Σ we obtain

$$\chi_{ij}(\beta) = \int_{-\infty}^{\beta\Delta_i^j} f(e^j)de^j. \tag{2}$$

Here the integral is (k–1) dimensional.

The maximum likelihood estimator for X is

$$L(\beta, \Sigma, y) = II_i II_j[\chi_{ij}(\beta)]^{y_{ij}}. \tag{3}$$

Unlike the usual model where the ϵ's are assumed to be iid, with zero expectation, we adopt the multinomial probit (MNP) assumption that f is the multivariate normal $N(0,\Sigma)$ with general variance-covariance matrix, Σ. (For a discussion of probit models, see Alvarez and Nagler, 1996.)

Markov Chain Monte Carlo (MCMC) techniques (Albert and Chib, 1993; McCulloch and Rossi, 1994) were used to estimate Σ. The details of our estimation results for the pure spatial model for the Netherlands and Germany are reported in Appendix A.

Figure 3 presents our estimate of (χ_{ij}) in the case of the Netherlands. Clearly, our estimation of χ_{ij} will depend on the ideal point of voter i and the location of party j. For example, a Dutch voter with an ideal point anywhere along the "social" axis and at the left extreme on the economic axis "will vote" for the PvdA with probability exceeding 0.9. On the other hand, any voter on the extreme right of the economic axis "will vote" for the VVD with certainty if near the libertarian extreme on the social axis and "will vote" for the CDA with probability about 0.8 if near the opposite end of the social axis. These probabilities make intuitive sense. To illustrate further, Figure 4 shows the distribution of ideal points of those voters who declared in the survey that they intended to vote for PvdA. The distribution of these voters has been normalized to sum to unity. Clearly the distribution is unimodal, with its mode near the PvdA position.

Obviously enough there are voters whose ideal points are nearer the D66 position for example, yet who intend to vote for the PvdA. Clearly a pure deterministic model of voting does not suffice. Our estimation would assign to a voter, with such an ideal point, probabilities of approximately .2 for D66, VVD, and CDA each, and 0.4 for PvdA. Such an assignment is not implausible.

Figure 5 shows a similar distribution for the ideal points of "CDU" voters in Germany in 1979. Appendix B presents our estimation results for a *joint electoral model* for the Netherlands and Germany, based on multinomial probit, using not only the spatial data but also individual characteristics of the

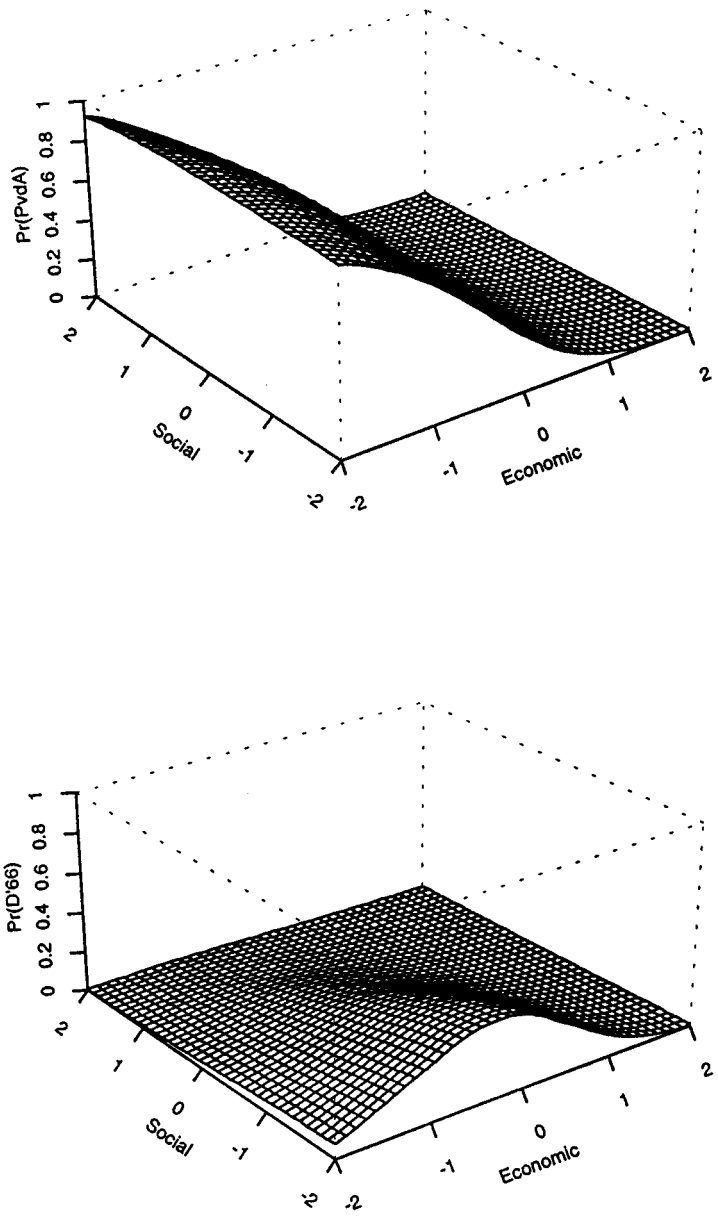

Figure 3a. Estimates of voter probabilities in the Netherlands for the PvdA and D66.

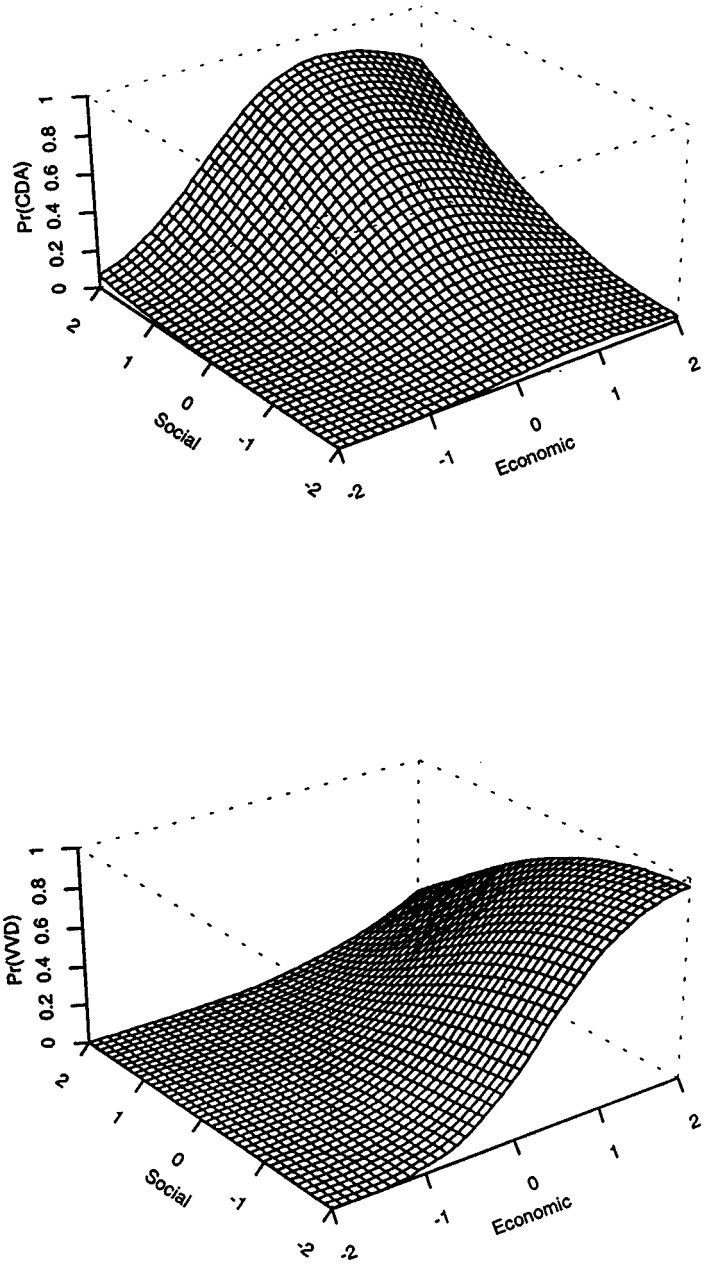

Figure 3b. Estimates of voter probabilities in the Netherlands for the CDA and VVD.

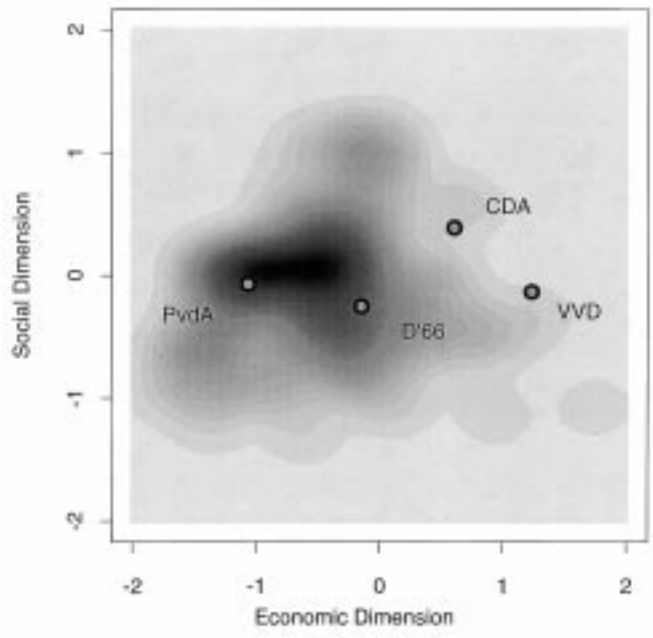

Figure 4. Dutch party and PvdA voter positions, 1979.

voters (derived from the Rabier, Inglehart, 1981, survey), such as occupation and income, religion, education, size of town of domicile, etc.

To briefly summarize the results, we find in both countries that the β-coefficient of the pure spatial model is statistically significant while the estimated covariance matrix is significantly different from the iid model. We also find that the pure spatial model is statistically superior to the joint model in the case of Germany. For the Netherlands, the joint electoral model is superior to the pure spatial model. We attempt to account for this result in the next section.

To illustrate the effectiveness of the pure spatial model, Table 5 compares the national, the sample, and the "estimated" vote shares for the four parties in the Netherlands. Since only four parties are considered in the model, we give approximate values for the share going to each party, computed as a percentage of the total national vote going to the four parties.

It is clear that the sample shares (based on n = 529) are quite close to the national shares for the 1977 and 1981 elections. The estimated vote shares are computed by finding the expected value, $E(\pi_j)$, of the vote for each party. Since vote shares are derived from (X_{ij}) and these are random variables, each vote share is also a random variable. Empirical histograms for the vote share of the four parties in the Netherlands are given in Figure 6.

The 95% confidence intervals on the estimated vote shares include the values for the sample shares for three of the four parties. (See Table N2 of the

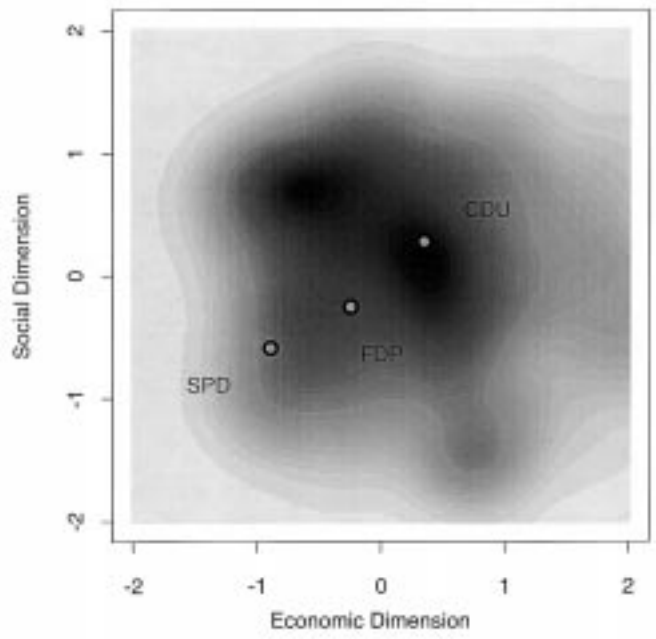

Figure 5. German party and CDU voter positions, 1979.

Table 5. Vote shares in the Netherlands' elections, 1977 to 1981

Party	Share of national vote[b]		Sample share in 1979[a]	Estimated share[c]	Optimal share
	1977 %	1981 %	%	%	%
D66	6.1	12.6	10.4	10.6	11 ± 4
PvdA	38.0	32.4	36.9	35.3	45 ± 3
CDA	35.9	35.2	33.8	29.9	40 ± 3
VVD	20.0	19.8	18.9	24.2	40 ± 3
	100.0	100.0	100.0	100.0	

[a] Based on Euro-barometer sample with n = 529.
[b] Vote shares are very close to seat shares, because of proportional representation. Shares are calculated on the basis of total vote to the four large parties, using *Keesing's Contemporary Archives.*
[c] Pure spatial model, as described in Appendix A.

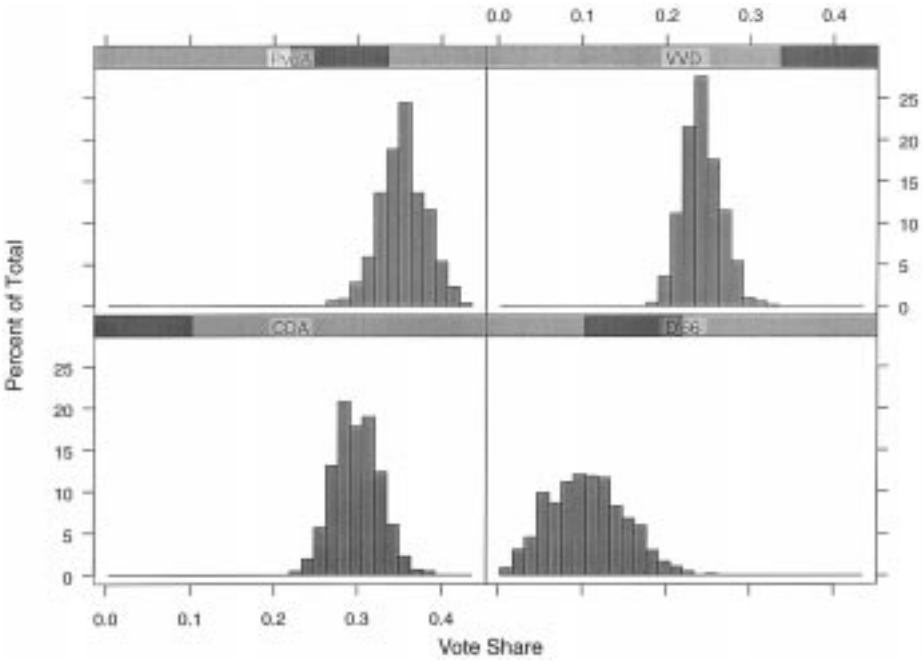

Figure 6. Histograms of estimated vote shares for four parties in the Netherlands (based on 1979 data).

Appendix for details.) The one anomaly to note is that, for the VVD, the national vote share (approximately 20%) and sample share (18.9%) lie outside the 95% confidence interval (20.8%, 28%) for the estimated expected vote share. However, the two empirical vote shares lie inside the 95% confidence interval of (14.9%, 24.6%) obtained from the joint model. The two models can be compared by considering Tables N2 and N3 in the Appendix.

Table 6 compares the national vote shares for the three parties in the 1976 and 1980 elections, together with the sample shares and our estimates, for Germany. Figure 7 gives the empirical histograms of the random variables π_j for each party in 1979 in Germany. The point to note is that 95% confidence intervals for expected vote shares under the pure spatial model for the three parties are approximately: CDU (.49, .58), SPD (.34, .46) and FDP (.01, .14). These confidence intervals are very similar to the confidence intervals for the joint model (compare Tables G2 and G3 in the Appendix).

Although these estimates do not provide much more information than the sample, they do allow us to draw conclusions about the electoral consequences were parties to adopt "strategic" policy positions. This we do in the next section.

Table 6. Vote shares in Germany, 1976 and 1980

Party	Share of national vote[b]		Sample share[a]	Estimated	Estimated
	% in 1976	% in 1980	in 1979, %	% share[c]	% share[d]
CDU	49.1	45.4	51.2	53.2	51.0
FDP	8.0	10.8	5.9	6.3	6.4
SPD	42.9	43.8	42.9	40.5	42.6
	100.0	100.0	100.0	100.0	100.0

[a] Based on Euro-barometer sample with n = 543 in 1979.
[b] Approximately 320,000 votes (out of over 37 million) for small parties are not included. These parties were not represented in the Bundestag. Vote shares are calculated as % of total vote to the three parties. In particular, the Greens gained 15% of the vote, but no seat in 1980.
[c] Based on the pure spatial model described in Appendix A.
[d] Using individual characteristics, as described in Appendix B.

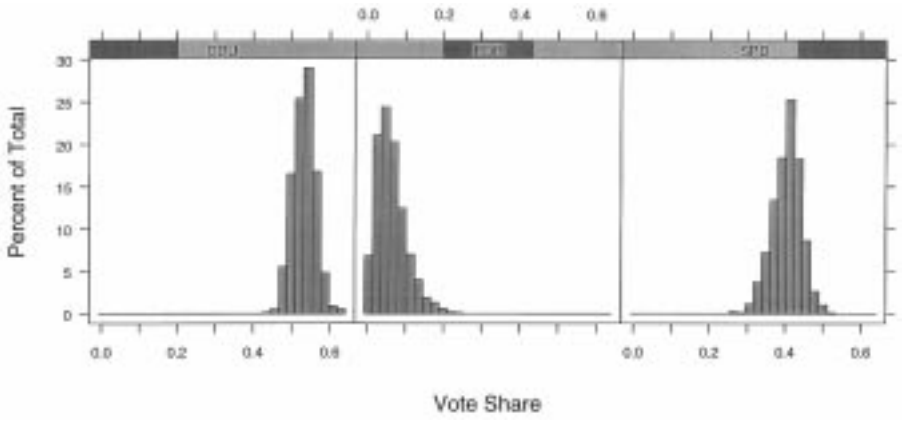

Figure 7. Histograms of estimated vote shares for the three parties in Germany (based on 1979 data).

It is worth noting, however, that the model we have constructed here, based as it is on the assumption of "stochastic voters", suggests that the underlying variation in vote shares is higher than might be supposed from simple analysis of electoral samples.

5. Models of party behavior derived from the analysis

As we have emphasized, party positions (in the model just described) were estimated by taking the median of party elite positions. Thus the positions of the parties are not the same as "declarations" to the electorate. Nonethe-

less, using these estimates of party positions, we found that the pure spatial MNP model of voting that we have just presented gives a plausible account of how Dutch and German voters respond to party positions. Thus we may assume that these party positions did have an effect on electoral response in the period in question. We now show that changing party positions would lead to an increase in expected party vote. Let us suppose that the elite members of the parties in question had beliefs concerning electoral response that are compatible with the spatial electoral model just discussed.

Consider the thought experiment where each party, j, considers a possible change in position from ϕ_j to a "strategic" position, ϕ^*_j, assuming all other party positions are fixed. Based on our MNP model, we compute the optimal ϕ^*_j, chosen as to maximize expected vote. In the Dutch case the optimal position for each party is at the electoral mean (that is, at the mean (0,0) of the sample ideal points). After such a move, our estimates of the vote increase by significant amounts. Table 5 shows the estimated optimal vote shares for the four Dutch parties. It indicates that the CDA, for example, could increase expected vote share by up to 10%, from about 30% to 40%. Had the CDA positioned itself at the electoral mean in 1977, we estimate that its expected number of seats would have been 55, rather than 49.[2]

Why do the parties not move to the electoral center, as predicted by the Downsian probabilistic model? To continue the thought experiment, suppose that during the 1977 election, the CDA positioned itself at the electoral mean, so that it gained 55 seats, instead of 49. Let us assume further that parties are committed to the positions they adopt. (We provide, below, an argument for this commitment.) We shall argue that this increase of seats for the CDA (coupled with the change of declared policy positions) would have adversely affected the final policy outcome, from the point of view of the CDA elite. To see this note that even with 55 seats, the CDA position at the mean, (0,0), is not a core position. In all probability, the CDA would have been obliged to form a coalition with the VVD. However, if the CDA were required to bargain from its declared position, then the policy compromise could in fact be further from its true policy position than the actual 1977 outcome. The same is true for the 1981 election situation. More importantly, given our estimate of the stochastic relationship between the CDA position and its share, there is no CDA position at which the CDA could have expected to gain a straight seat majority (given our confidence intervals). Moreover, the probability that the CDA could have occupied the core is essentially zero.

In the same way, a thought experiment involving the PvdA suggests that it could have won approximately 60 (± 4) seats in the 1981 election, by moving to the origin. If the PvdA were obliged to commit to that position, then it would indeed be at a core with high probability and possibly able

to form a minority government. Now compare the situation in 1981 if the PvdA declares its "true" position in contrast to the outcome resulting from the PvdA at a core position at the origin. In the former case there is a significant probability of a {PvdA,D66,CDA} coalition in which the PvdA can presumably affect policy. On the other hand, at the core the PvdA may be able to implement a minority government, control all the perquisites, but be obliged to implement a policy far from its ideal position. It is not implausible that the PvdA party elite would prefer to declare the party's "true" position (we explore this somewhat further below).

In a situation such as we have described in the Netherlands, we suggest that there was an insignificant probability that any party could occupy a post-election core position and implement a policy that it found preferable to the lottery resulting from a true declaration of preference.

The case in Germany is somewhat different. As we have seen, the FDP is "almost" at the core position and has been in almost every government for decades. It could be dislodged from this position were the SPD to move sufficiently close to the FDP. However, assuming policy commitment again, this would have obliged the SPD to implement a policy that the elite of the party would have found highly undesirable.

We infer that if a party adopts a policy position simply to increase the number of seats that it controls, then this may not increase the power that the party has to implement desired policy. The willingness of a party to adopt vote maximizing policies depends, therefore, on the trade-off in its "utility" function between the perquisites of office and the importance of policy. However, to determine the optimal position, it would appear that the party must compute not just the effect of a change of position on its vote share, but must also consider how that change affects the post-election lottery over the choice of the coalition government. If there is no policy change that leads to a potentially "dominant" party being at a core position, with significant probability, then there are plausible arguments to be made that the declarations of a "true" policy preference is "rational" for such a party.

To make this argument clearer, we need to examine in more detail our notion of the preferred point, ϕ_j, of party j. In our estimation, we chose each party's point to be at the median (in both dimensions) of the ideal points of the party elite (using the ISEIUM, 1983, responses of party delegates). To illustrate, Figure 8 represents our estimate of the density function of the ideal points of the PvdA delegates. Comparing Figure 8 with Figure 4 (showing PvdA voter ideal positions), it is clear that the PvdA delegate variance is much less than PvdA voter variance. It is reasonable to suppose that the PvdA has some leeway in declaring a pre-election position to the electorate. However, it is also obvious that the party cannot choose *any* position at will. If we view the

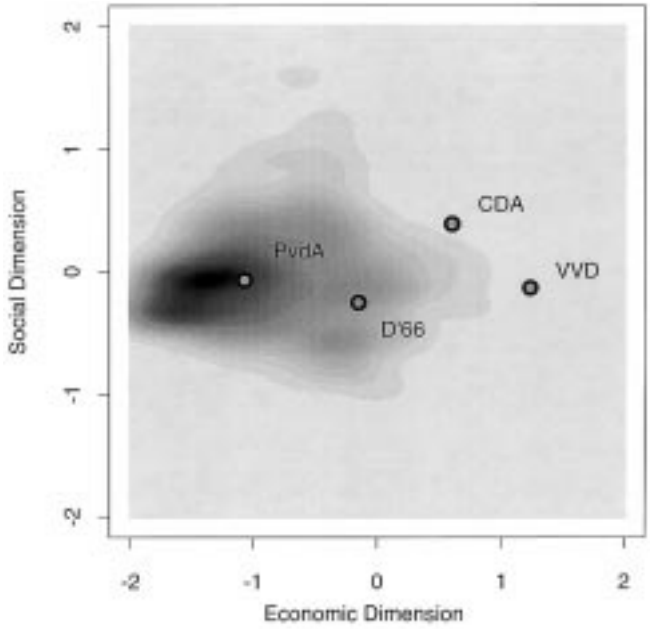

Figure 8. Empirical distribution of estimated ideal points of PvdA delegates (based on 1979 Dutch data).

PvdA delegates as a committee, then it is almost certain that an internal PvdA core will exist (under majority decision-making by the delegates). This "internal" party core will lie at the (two-dimensional) median of the delegates' ideal points. It is therefore sensible to regard this "internal" core position as the party's ideal point, as we have done in the estimation just discussed.[3]

Indeed, we can regard this internal party core point as the *sincere* choice of the party. Now consider internal party decision-making over the choice of a policy point to declare to the electorate. Each delegate of the party has an ideal policy point, as well as beliefs about electoral response and the nature of post-election negotiation. Thus, each delegate will have a personal choice for the best response the party can make to the policy positions of the other parties. It is reasonable to suppose that a delegate's "best response" is continuous in the delegate's ideal point. Since there is a "sincere" internal party core (under majority rule among the delegates), there will also be an unbeaten choice among the party delegates for the policy point to offer to the electorate. Such a point we can refer to as the *strategic* choice of the party. Clearly, the party may commit to this position by choosing the leader of the party to be exactly that delegate whose ideal point coincides with the strategic choice of the party. Now, consider a situation where each party makes a strategic best response to the declarations of the other parties. Since the strategic best response will

Table 7. Maximizing expected vote

Party	Sample share	"Ideal" point	"Strategic" point	Estimated vote	95% C.I.
D66	10.4	(–0.1, –0.2)	(–0.1, –0.2)	10.8	(4.5, 16.5)
PvdA	36.9	(–1.1, +0.0)	(–1.1, +0.0)	36.4	(33.0, 40.0)
CDA	33.8	(+0.6, +0.4)	(+0.6, +0.4)	32.8	(28.0, 37.0)
VVD	18.9	(+1.2, –0.3)	(+1.5, –1.0)	20.0	(16.5, 23.5)

be continuous in the other parties' positions, standard arguments can be used to demonstrate existence of a mutual strategic best response, or Nash equilibrium. (See Schofield, 1996, for an outline of such an existence proof.) Notice that this pre-election Nash equilibrium in party positions is credible after the election, since each party position will be associated with the ideal point of a member of the party elite. Moreover, this member, the "leader" of the party, will have every incentive to bargain effectively with other party leaders in post-election coalition negotiations.

Let us continue with this thought experiment and suppose that the four parties in the Netherlands had adopted positions that were strategic Nash equilibria with respect to their beliefs about the electoral response to these positions, circa 1979–81, and also their beliefs about post-election bargaining. Assume further that the parties' electoral beliefs are generally compatible with the model of stochastic voting that we have constructed. If we modify the parties' positions to bring the model's estimated expected vote shares into line with the 1981 election results, then we have a method to estimate the parties' declared Nash equilibrium position. Since we have observed an anomaly in the expected vote share of the VVD, as estimated by our MNP model, this suggests that we consider a position for the VVD that differs from this party's sincere position. If we keep the three other parties at their sincere positions and move the VVD from its sincere position (+1.2,–0.3) to (+1.5,–1.0), then (as Table 7 reports) the estimated expected vote share of the VVD drops from 24.2% to 20.0% and the 95% confidence interval for the party's vote share changes from (20.8%,28%) to (16.5%,23.5%). This interval now includes the sample share and the national vote shares of the VVD in the 1977 and 1981 elections. The estimated expected vote shares of the other three parties are still close to the sample shares.

Of course we have no direct evidence that the VVD did indeed declare a policy position to the electorate in the 1977 and 1981 elections that was more radical than its sincere position.

In our view, the choice between the joint electoral model (as described in Appendix B) and the pure spatial model (incorporating the VVD strategic position) depends on the plausibility of a theoretical argument about why a party such as the VVD could rationally choose to declare a policy point which is more radical or extreme than its sincere internal core position.

Appendix C gives a stylized argument, adapted from Schofield (1993) and Schofield and Parks (1996), to show that if post-election coalition formation is represented as a lottery, then a party such as the VVD could indeed choose a more extreme position. The point to note about this model of party calculation is that although the VVD may believe that it would lose a few percent of the vote by declaring a more radical position, it could also plausibly believe that it would, in expectation, pull the policy compromises made either with the PvdA or CDA towards its internal core position.

The suggestion that the VVD adopted a more extreme position also allows us to make sense of the 1977 attempt by the CDA to form a coalition with the PvdA. As Figure 1 illustrates, the {CDA,VVD} internal core points are nearer to each other than the {CDA,PvdA} internal core points, and so the former coalition might appear more likely. If, however, the VVD had adopted the more extreme position, as we have suggested, so that a {CDA,VVD} compromise would favor the VVD, then it would indeed be rational for the CDA to approach the PvdA first.

Of course, such a strategic declaration by the VVD is inherently risky. Indeed, it may have opened the way for smaller parties to enter the electoral fray in 1981, leading to the loss of the {VVD,CDA} majority, and to the diminution of the attractiveness of the VVD as a coalition partner.

Having argued that our analysis suggests why the VVD may have declared a more extreme position, we can also add somewhat to our earlier suggestion that the PvdA and CDA would be likely to declare their true, or internal core, positions. For example, in the case of the PvdA, the more centrist delegates of the party would presumably prefer a move by the party to the core position, hoping for a minority PvdA government. On the other hand, more extreme delegates of the party would prefer a leader of the party to adopt a position on the left of the economic dimension, hoping to influence policy through coalition negotiation.

For both the PvdA and CDA we might expect considerable internal disagreement within these parties, resulting however, in policy declarations chosen to be compromises close to the sincere policy points of the two parties.[4]

The general model we propose emphasizes the heterogeneity of ideal policy points within each party, j, together with the selection of a leader,

whose ideal point, z_j, coincides with the majority choice of an equilibrium declaration, ϕ_j^*, made by the party elite.

6. Conclusion

Models of elections have generally been based on the "Downsian" assumption of expected vote maximization. The conclusions of such models are at variance with the observation that parties do not converge to an electoral center.

The empirical work presented here suggests that a pure spatial electoral model is quite satisfactory in modeling electoral response to party positions. The estimation allows us to determine approximately how much party vote shares would increase as a result of policy convergence to the electoral mean. Although nominal political power may be increased by such a move, we argue that the power to implement desired policies need not increase.

For the Netherlands we suggest that the electoral model allows us to distinguish between sincere and strategic party positions. In particular, we infer that the two larger parties adopted policy positions that were close to their sincere, or internal core, positions. We have also provided an argument to explain why a small, non-centrist party such as the VVD, could reasonably adopt a policy position that is more extreme than its sincere policy position. In Germany, on the other hand, we infer that the balance of preferences within each party leads to the declaration of a policy point close to the sincere, internal core, of the party. For both countries, our empirical and theoretical modeling provides an explanation for the evident fact that parties facing proportional representation systems do not converge to the electoral center.

We argue that understanding political choice in multiparty democracies can be based on the following research program: (a) build a stochastic electoral model, of the kind we have constructed here, which can be used as a proxy for the beliefs of the party elite, (b) assume that each party's declaration is associated with the position of a member of the party elite, who then bargains with other party representatives using the post-election party strength as a political resource, (c) model the post-election coalitional bargaining game in terms of a lottery of expected coalition outcome inside the post-election heart, (d) solve the internal pre-election party negotiation game over the choice of party representative and party declaration.

We intend to develop this framework in future research, by rejecting the usual assumption that each party is a unitary actor, and by studying the relationship between party declarations (or manifestos) and the distribution of elite ideal points. We also hope to relate this model of internal party choice

to current work on post-election coalition negotiation (Schofield, Sened, and Nixon, 1997).

We suggest that the combination of stochastic electoral models of elections together with spatial committee analysis can provide a theoretically powerful, and substantively relevant way to understand party dynamics in multiparty polities based on proportional representation. Although plurality electoral systems are more difficult than proportional systems to analyse, it should be possible, in principle at least, to extend the analysis to study strategic behavior in Westminster style polities. Finally, since the underlying model involves delegates, it should also be possible to deal with factional polities without disciplined parties.

By concentrating on elite political actors, instead of parties, it may be possible to begin to consider the question of the formation of elite groups and the building of political parties. By this method we may gain a better understanding of the differences between the various types of polities mentioned in the introduction, and contribute to the development of a formal theory of comparative politics.

Notes

1. The effective number is the inverse of the Herfindahl index of concentration.
2. Note that the "electoral heart" of the sample, and presumably of the entire electorate, is a very small set lying close to the origin (0,0). The relationship between the "electoral heart" and the "political heart", as generated by party positions and weights, is one way to characterize a multiparty system.
3. Even if the internal PvdA core is empty, the "heart" of the PvdA delegates, viewed as a committee, will be a very small set centered on this two-dimensional median.
4. Of course, one could fine-tune the electoral model to equate all expected vote shares with sample vote shares, leading us to infer that all parties "strategize". This would seem somewhat pedantic, given the margins of error. The indirect evidence for VVD strategizing persists even if we use the delegates' mean position rather than the median.

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Appendix A: Spatial model

The Euro-barometer questionnaire (Rabier and Inglehart, 1981) asked seven questions covering topics such as income distribution, terrorism, nuclear energy, public control of enterprises, environmental protection, control of multinational corporations and abortion. Seven similar questions were asked of political elites of known party allegiance (through the Institut fur Sozialwissenschaft and the Europa Institute, Universitat Mannheim, 1983). Exploratory factor analysis uncovered two factors in both the Netherlands and Germany. Tables N-1 and G-1 give the factor

Table N-1. Factor loadings, The Netherlands

Issue	Dimension 1	Dimension 2
Income distribution	+.510 (10.86)	–0.148 (1.92)
Terrorism	–0.232 (4.28)	–0.253 (2.51)
Nuclear energy	–0.297 (6.74)	–
Enterprises	+0.526 (12.0)	–
Environment	+0.306 (7.46)	–
MNC	+0.612 (12.6)	–0.229 (2.42)
Abortion	+0.327 (5.56)	+0.39 (2.45)

Chi-square over d.o.f. = 1.76. Sample size (n) = 529.

Table G-1. Factor loadings, Germany

Issue	Dimension 1 Economic	Dimension 2 "Corporatism"
Income distribution	.331 (5.8)	+0.156 (2.51)
Terrorism	0.351 (6.2)	–0.273 (3.73)
Nuclear energy	–	–0.641 (4.78)
Environment	0.668 (9.1)	–0.173 (2.62)
MNC	0.343 (5.71)	+0.222 (3.26)
Abortion	0.283 (5.24)	–

Chi-square over d.o.f. = 3.11. Sample size (n) = 543.

loadings on the various issues as follows (t-values in parentheses). In Germany, the question on enterprises was redundant.

Responses to the seven questions (ranging from strongly favor (1) to strongly oppose (4)) were renormalized to have mean 0, standard deviation 1.

The factor loadings were then used to generate scoring coefficients for each of the responses for each country. These were combined to give for each voter an ideal point on the two dimensions. The same technique was employed for the political elites. Knowing party allegiance we computed the median of the elite for each party on each dimension. This was used as our estimator for each party's ideal or "sincere" policy point.

For reasons of identification of the estimation for the Netherlands we adopted a variant of the probabilistic model, using the D66 party as a baseline. That is, letting ϕ_4 be the estimated position of the D66 party we assumed that realized utility of

voter i for party j, given (ϕ_1,...,ϕ_4), was

$$\begin{aligned} u_{ij}(\phi) &= \beta_0 - \beta(\delta_{ij})^2 + \beta(\delta_{i4})^2 + \epsilon_j \text{ for } j = 1, 2, 3, \text{ and} \\ u_{i4}(\phi) &= \epsilon_4. \end{aligned}$$

Just as in Section 4, Prob ($u_{ij} > u_{il}$ for all $l \neq j$) = Prob($e_j < \beta\Delta^j{}_i$). Under the usual assumption of *independence* of errors it is an easy matter to show that the variance (ϵ_l–ϵ_j) = variance (ϵ_l) + variance(ϵ_j) while the covariance terms are cov(ϵ_l–ϵ_j,ϵ_m–ϵ_j) = variance (ϵ_j). In our analysis we used a statistical routine involving Markov Chain Monte Carlo Simulation and data augmentation, due to Chib and Greenberg (1996), that did not assume iid errors.

(Technical details can be found in Quinn, Martin, and Whitford, 1997, and Martin and Quinn, 1997).

To present the results for the Netherlands, we use the code {PvdA,VVD,CDA,D66} ≡ {1,2,3,4}. We define the covariance terms σ_{lm} = cov(ϵ_l–ϵ_4,ϵ_m–ϵ_4) and the variance terms $\sigma^2{}_{ll}$ = var(ϵ_l–ϵ_4).

Table N-2 presents our estimate for these variance, covariance terms, together with the estimated coefficients β_0 (constant) and β (spatial distance).

Note that the model is unchanged if all utility terms are multiplied by a constant. Thus, to identify the model we assume $\sigma^2{}_{11}$ = 1.0, and present the variance and covariance estimates in terms of $\sigma^2{}_{11}$.

The logarithms of the marginal likelihood for these pure spatial models are –545 (for the Netherlands) and –506 (for Germany). The Bayes' factors (Kass and Raftery, 1995) for comparing these models with a null hypothesis (that X is not determined by the matrix δ) gives vanishingly small probability to the null hypothesis.

The estimated 95% confidence intervals strongly suggest that the coefficients, β_0 and β, are significantly non-zero. Moreover, confidence intervals on the covariance matrix indicate that the covariance terms are generally not half variance terms (as would be true under iid).

To present the estimation for Germany, we use the code {SPD,CDU,FDP} = {1,2,3} and normalize with respect to the FDP. Table G-2 presents our findings.

Again $\sigma^2{}_{11}$ is set at 1.0. Clearly the two variance terms ($\sigma^2{}_{11}$ and $\sigma^2{}_{22}$) are dissimilar. It is possible, but not probable, that the covariance term σ_{12} is 0.5.

Appendix B: Individual factors

We also estimated a *joint* model utilizing the spatial matrix (δ_{ij}) but involving individual and demographic characteristics that were recorded in the Rabier-Inglehart Survey.

Table N-2. Estimation results for the Pure Spatial Model in the Netherlands

Parameter	Party	Posterior mean	SD	95% CI Lower	95% CI Upper
Spatial distance		0.456	0.120	0.230	0.710
Constant		0.512	0.064	0.391	0.638
Variance-covariance estimates					
σ^2_{11}		1.000	0.000	1.000	1.000
σ_{12}		–0.376	0.261	–0.769	0.084
σ_{13}		–0.107	0.188	–0.426	0.224
σ^2_{22}		1.012	0.289	0.618	1.546
σ_{23}		0.384	0.200	0.092	0.752
σ^2_{33}		0.696	0.248	0.360	1.134
Sample vote share	D66	0.104			
	PvdA	0.369			
	VVD	0.189			
	CDA	0.338			
Predicted vote share	D66	0.106	0.044	0.038	0.182
	PvdA	0.353	0.027	0.309	0.397
	VVD	0.242	0.022	0.208	0.280
	CDA	0.299	0.027	0.255	0.342

Table N3 records the effect of incorporating these additional characteristics. As the table shows, estimated vote shares for D66 and PvdA hardly shift. The high estimated value of the VVD share now drops from 24.2% to 19.5% (close to the sample share of 18.9%), while the low share for the CDA of 29.9% climbs to 34.9% (close to the sample share of 33.8%).

The logarithm of the marginal likelihood increases from –545 to –515.

To interpret the increase of log likelihood as we change from the pure spatial model, to the joint model, we can compute the Bayes' Factor (Kass and Raftery, (1995) between the two models. This is simply the ratio of the marginal likelihoods of the joint and spatial models, and can be computed as exp (–515 + 545) = exp (30) > 10^{13}. This suggests that the joint model is superior, in a statistically significant sense, to the pure spatial model. As a comparison of the confidence intervals on the VVD vote share suggests, the joint model is statistically superior to the pure spatial model in capturing voting behavior in the Netherlands.

Table G3 records the effects of incorporating the individual and demographic characteristics in the model for Germany. Clearly, expected vote shares become close

Table G-2. Estimation results for the Pure Spatial Model in Germany.

Parameter	Party	Posterior mean	SD	95% CI Lower	95% CI Upper
Spatial distance		0.239	0.066	0.120	0.377
Constant		1.078	0.228	0.604	1.509
Variance-covariance estimates					
σ^2_{11}		1.000	0.000	1.000	1.000
σ_{12}		0.025	0.431	–0.644	0.727
σ^2_{22}		3.735	2.009	1.361	7.942
Sample vote share	FDP	0.059			
	SPD	0.429			
	CDU	0.512			
Predicted vote share	FDP	0.063	0.039	0.013	0.136
	SPD	0.405	0.038	0.337	0.462
	CDU	0.532	0.029	0.486	0.578

to the sample shares. In particular the SPD share increases from 40% to 42.6% (close to the sample of 42.9%).

The logarithm of the marginal likelihood drops from –506 to –623.

The Bayes' Factor between the spatial and joint models for Germany is now exp (–506 + 623) = exp (117) $> 10^{50}$. This suggests that the pure spatial model is statistically superior to the joint model in Germany, at least in the period in question. This is intuitively obvious from the fact that all sample vote shares lie within the 95% confidence intervals of the predicted vote share.

Appendix C: Strategic party choice

Here, we briefly outline the model of post election coalition bargaining, proposed in Schofield and Parks (1996). Suppose that three parties {A,B,C} have the ideal points presented in Figure 9, and that the parties are committed to their declarations (in the manner proposed in the text). For ease of presentation suppose that they believe that, whatever their declarations, the electoral response will result in party strengths such that any pair of parties will control a majority. If the parties' declarations are

Table N-3. Estimation results for the Joint Model, the Netherlands

Variable	Party	Posterior mean	SD	95% CI Lower	95% CI Upper
Spatial distance		0.499	0.065	0.376	0.627
	PvdA	0.926	0.252	0.444	1.421
Manual labor	VVD	–0.714	0.456	–1.699	0.103
	CDA	0.239	0.264	–0.274	0.777
	PvdA	–0.060	0.082	–0.225	0.101
Religion	VVD	–0.112	0.114	–0.353	0.092
	CDA	0.504	0.098	0.339	0.735
	PvdA	–0.012	0.022	–0.054	0.032
Income	VVD	0.049	0.028	–0.001	0.108
	CDA	–0.010	0.021	–0.052	0.031
	PvdA	0.382	0.103	0.179	0.582
Town size	VVD	–0.131	0.137	–0.427	0.111
	CDA	–0.046	0.113	–0.279	0.158
	PvdA	–0.083	0.029	–0.141	–0.028
Education	VVD	–0.007	0.038	–0.080	0.067
	CDA	–0.077	0.028	–0.140	–0.026
Constant		0.256	0.239	–0.185	0.767
Variance-covariance estimates					
σ^2_{11}		1.000	0.000	1.000	1.000
σ_{12}		–0.350	0.265	–0.772	0.104
σ_{13}		–0.065	0.193	–0.386	0.252
σ^2_{22}		1.782	0.695	0.933	2.948
σ_{23}		0.564	0.309	0.161	1.107
σ^2_{33}		0.671	0.233	0.380	1.067
Sample vote share	D66	0.104			
	PvdA	0.369			
	VVD	0.189			
	CDA	0.338			
Predicted vote share	D66	0.108	0.030	0.061	0.161
	PvdA	0.347	0.026	0.306	0.389
	VVD	0.195	0.028	0.149	0.246
	CDA	0.349	0.030	0.301	0.401

Table G-3. Estimation results for the Joint Model, Germany

Parameter	Party	Posterior mean	SD	95% CI Lower	95% CI Upper
Spatial distance		0.295	0.079	0.154	0.466
Manual labor	SPD	0.358	0.275	–1.174	0.910
	CDU	–0.269	0.467	–1.251	0.606
Religion	SPD	–0.341	0.467	–0.641	–0.015
	CDU	0.946	0.393	0.312	1.827
Income	SPD	–0.023	0.028	–0.078	0.030
	CDU	–0.056	0.047	–0.154	0.033
Town size	SPD	0.096	0.127	–0.154	0.347
	CDU	–0.594	0.247	–1.146	–0.165
Education	SPD	–0.068	0.043	–0.015	0.015
	CDU	–0.117	0.082	–0.286	0.037
Constant		1.843	0.474	0.978	2.835
Variance-covariance estimates					
$\sigma^2{}_{11}$		1.000	0.000	1.000	1.000
σ_{12}		0.035	0.379	–0.584	0.670
$\sigma^2{}_{22}$		7.292	4.106	2.442	14.936
Sample vote share	FDP	0.059			
	SPD	0.429			
	CDU	0.512			
Predicted vote share	FDP	0.064	0.026	0.026	0.111
	SPD	0.426	0.033	0.368	0.481
	CDU	0.510	0.029	0.462	0.558

sincere, then coalition policy outcomes will be a lottery across the "heart" ABC. Let U(ABC) be the von Neumann Morgenstern utility of party C for this lottery, where U is derived from the underlying Euclidean preferences of the party. Consider a possible declaration by party C of the position C′, chosen such that the triangles {C′,C,B}, {C′,C,A} and {A,B,C} are all equivalent. By symmetry each of these lotteries occurs with probability, $\frac{1}{3}$, so the von Neumann utility of party C for the lottery across ABC′ can be represented as

$$U(ABC') = \frac{1}{3}U(ABC) + \frac{1}{3}U(C'CB) + \frac{1}{3}U(C'CA).$$

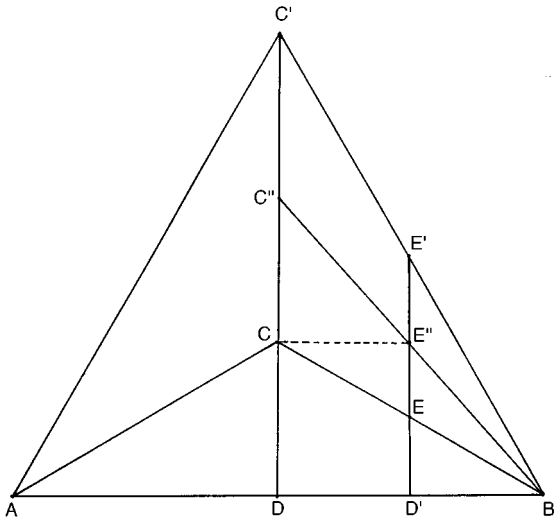

Figure 9. Strategic party choice

By symmetry again U(ABC)=U(C′CB) = U(C′CA), so U(ABC′) = U(ABC).

If the lotteries are continuous in the declarations, and U is continuous then there exists a point C″ on the arc [C,C′] which maximizes U. This point C″ is the best response of party C to the declarations, A and B. Similar arguments show existence of a Nash equilibrium, namely mutual best responses of A″ by A to B″, C″, etc.

Obviously, the best response C″ by C depends on the assumptions made on the nature of the lottery. Schofield and Parks (1996) have made specific assumptions on the nature of this lottery, which permit computation of best response. To illustrate suppose after the declarations {A,B,C}, that coalition {A,B} chooses as a policy compromise the mid-point $\frac{1}{2}(A + B)$ = D, etc., and that coalitions occur with probability inversely proportional to the distance between coalition members. Thus, when party C declares position C, then coalition {C,B} chooses the midpoint E = $\frac{1}{2}$(B+C), and when party C chooses C′, then the same coalition {C,B} chooses E′ = $\frac{1}{2}$(C′+B). It is evident from Figure 9 that the best response by C to {A,B} is a point C″, such

that the midpoint $E'' = \frac{1}{2}(C''+B)$ minimizes the distance between C and the arc EE'. Thus best response by C is to adopt a more extreme position than its ideal point.

Under the above assumptions on the coalition lottery, Schofield and Parks show that with the given configuration of ideal points, parties A and B will converge to one another in Nash equilibrium, while C will "diverge." If non-policy portfolios are added to the calculations, the extent of convergence by A and B will increase, while C's divergence will decrease.

To extend this model to the case of heterogeneous preferences within party C, for example, note that any delegate of the party with an ideal point in a neighborhood of C will have a best response that is further from the line AB than is their ideal point. We suggest that such a model of best response accounts for our empirical inference that the VVD adopted a more extreme strategic position than its ideal, or internal core, point.

Computation of best response and of Nash equilibrium is highly nonlinear and can only be analytically computed in simple symmetric situations such as described by Figure 9. In general, asymmetries in the configuration of the parties' sincere choices will become even more exaggerated when the parties adopt best responses to each others' positions. These inferences appear to be robust with respect to the specific assumptions made about the coalition lotteries.